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BRANCHING METHODS OF ANALYZING A DISTURBANCE OF THE CRITICAL-PRESSURE SPECTRUM OF SHELLS OF REVOLUTION AND SOME APPLICATIONS OF THESE METHODS

V. V. Larchenko

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New phenomena in the branching loss of stability of an elastic shell were discovered in a study conducted for a singular perturbation. It was found in particular that disturbance of the middle surface and the load is accompanied by a change in the type of bifurcation of the branch points, the rotation group of the minor equilibrium mode, and the multiplicity of the eigenvalues. Conditions were formulated for the functionals of the branching equation for which the multiplicity increases to the specified value. This makes it possible to significantly simplify the theory of models of instability. To establish the above facts, it is important that the spectrum be crowded at $\mu \to 0$ (μ is a natural small parameter with higher derivatives). A theoretical-empirical method of evaluating the effectiveness of electrophysical loading of thin shells was proposed within the framework of the completed study.

1. Let (r, φ) be a polar coordinate system whose origin is located at the tip of a shallow spherical segment. We will examine the below Marguerre–Vlasov problem [1-3] in the space

$$\begin{gathered}
\mu^{2(3-k)}\Delta^2 w = \theta\Delta\Phi + \mu^q[L(w, \Phi) + L(w_\tau, \Phi)] + \mathscr{P}(r),\ (r, \varphi) \in \Omega,\\
\mu^{2(k-1)}\Delta^2\Phi = -\theta\Delta w - \mu^q[L(w, w) + 2L(w, w_\tau)]/2,\\
w = w' = 0,\ A\Phi = B\Phi = 0,\ r \in \partial\Omega,\\
rL(u,\ v) = u''Av + v''Au - 2r^{-1}BuBv,\ \ A(\cdot) = (\cdot)' + r^{-1}(\cdot)^{\cdot\cdot},\\
B(\cdot) = [(\cdot)' - r^{-1}(\cdot)]^{\cdot},\ \mathscr{P}(r) = p + \delta\eta(r),\ p \in \{p_n\},\ |\delta| \ll 1.
\end{gathered} \tag{1.1}$$

Here, w is the normal displacement of the middle surface; Φ is the Airy stress function; $\mathscr{P}(r)$ is the external pressure; $\{p_n\}$ is a sequence of critical pressure of a perfectly spherical dome; δ is the density of the pressure disturbance; $\mu^2 = h/a\gamma$ is a natural small parameter; h is the thickness; $2a = \operatorname{diam}\Omega$; $\gamma^2 = 12(1-\nu^2)$; $\nu \in (0,\ 0,5)$ is the Poisson's ratio; θ is a half-angle; $w_\tau(r, \varphi)$ is a 2π-periodic disturbance of the middle surface such that $w_\tau(r,\ \varphi) \in \mathscr{F}_\tau(\Omega)$, $|\tau_n| \ll 1$, $\|f_n(r)\|_c = 1$, where

$$\mathscr{F}_\tau(\Omega) = \left\{f_\tau \in \mathring{H}^2(\Omega)\,|\,f_\tau(r, \varphi) = \sum_{n=1}^{\infty} f_n^\tau(r)\cos n\varphi,\quad f_n^\tau(r) = \tau_n f_n(r)\right\}.$$

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If the solution of problem (1.1) is known, then the transition to dimensional variables is made by means of the formula

$$F = \mu^{2(k-1)+q} E a^2 \Phi, \ \{W, W_\tau\} = a\mu^q \{w, w_\tau\},$$
$$R = ar, \ \{Q_p, \rho_\delta\} = \mu^{2(k-1)+q} E\gamma \{p, \delta\}. \tag{1.2}$$

Here and below, k and q are numerical parameters for which the sought unknowns in (1.2) have the order O(1) at $\mu \to 0$.

2. Projecting the perturbation of the solution in eigenvalues of problem (1.1), we follow [4] and introduce the small parameter ξ_n. We represent the vector-function (w, Φ) in the form of a Poincaré–Lyapunov series in the neighborhood of a simple nonaxisymmetric branch point:

$$w \sim \int_1^r \omega(z)\,dz + \xi_n r^n \omega_n(r) \cos n\varphi + \xi_n^2 \left[\int_1^r g_n(z)\,dz + r^{2n}\gamma_n(r) \cos 2n\varphi \right] + \ldots,$$
$$\Phi \sim \int_0^r f(z)\,dz + \xi_n r^n \varphi_n(r) \cos n\varphi + \xi_n^2 \left[\int_0^r \psi_n(z)\,dz + r^{2n}\delta_n(r) \cos 2n\varphi \right] + \ldots \tag{2.1}$$

Using (2.1), we reduce the Marguerre–Vlasov system to a recurring sequence of boundary-value problems. The zeroth approximation for ξ_n:

$$\mu^{2(3-k)} \mathscr{A}\, \omega = \theta r f + \mu^q \omega f + (1/2) p r^2, \ \mu^{2(k-1)} \mathscr{A}\, f =$$
$$-\theta r \omega - (1/2)\mu^q \omega^2, \tag{2.2}$$
$$\omega(1) = f(1) = 0, \ \lim_{r\to 0} |\omega r^{-1}| < \infty, \ \lim_{r\to 0} |f r^{-1}| < \infty, \ \mathscr{A}(\cdot) = r \frac{d}{dr} r^{-1} \frac{d}{dr} r(\cdot).$$

The nonlinear eigenvalue problem for the parameter p has the form

$$\mu^{2(3-k)} \Delta_n{}^2 \omega_n = \theta \Delta_n \varphi_n + \mu^q [L_f{}^n \omega_n + L_\omega{}^n \varphi_n],$$
$$\mu^{2(k-1)} \Delta_n{}^2 \varphi_n = -\theta \Delta_n \omega_n - \mu^q L_\omega{}^n \omega_n, \tag{2.3}$$
$$r = 1, \ \chi_n = \chi_n' = 0, \ \lim_{r\to 0} |\chi_n' r^{-1}| < \infty, \ \lim_{r\to 0} |\chi_n''' r^{-1}| < \infty, \ \chi_n = \omega_n, \varphi_n,$$

where $\Delta_n(\cdot) = (\cdot)'' + (2n+1) r^{-1} (\cdot)'$; $L_u^n(\cdot) = r^{-1} u' \left[(\cdot)' + (n - n^2) r^{-2} (\cdot) \right] + r^{-1} u \left[u'' + 2n r^{-1} (\cdot)' + n r^{-2} (n - 1)(\cdot) \right]$.

We write the second approximation of ξ_n for the axisymmetric components as

$$\mu^{2(3-k)} \mathscr{A}\, g_n = \theta r \psi_n + \mu^q [\omega \psi_n + g_n f] + \Omega_n(\omega_n, \varphi_n),$$
$$\mu^{2(k-1)} \mathscr{A}\, \psi_n = -\theta r g_n - \mu^q \omega g_n - (1/2)\Omega_n(\omega_n, \omega_n),$$
$$\Omega_n(\omega_n, \varphi_n) = (1/2)\mu^q r^{2n} [\sigma_n \omega_n \sigma_n \varphi_n - n^2 r^{-1} (\varphi_n \sigma_n \omega_n + \omega_n \sigma_n \varphi_n) +$$
$$n^2 r^{-2} \omega_n \varphi_n], \ \sigma_n(\cdot) = n r^{-1}(\cdot) + (\cdot)', \tag{2.4}$$
$$g_n(1) = \psi_n(1) = 0, \ \lim_{r\to 0} |r^{-1} g_n| < \infty, \ \lim_{r\to 0} |r^{-1} \psi_n| < \infty.$$

The boundary-value problem for the nonaxisymmetric components of the Poincaré–Lyapunov approximation is as follows:

$$\mu^{2(3-k)} \Delta_{2n}^2 \gamma_n = \theta \Delta_{2n} \delta_n + \mu^q \left[L_\omega^{2n} \delta_n + L_f^{2n} \gamma_n \right] + \alpha_n(\omega_n, \varphi_n),$$
$$\mu^{2(k-1)} \Delta_{2n}^2 \delta_n = -\theta \Delta_{2n} \gamma_n - \mu^q L_\omega^{2n} \gamma_n - (1/2)\,\alpha_n(\omega_n, \omega_n),$$
$$2\alpha_n(\omega_n, \varphi_n) = (\vartheta_n \omega_n s_n \varphi_n + s_n \omega_n \vartheta_n \varphi_n + 2n^2 r^{-2} m_n \omega_n m_n \varphi_n)\mu,$$
$$\vartheta_n(\cdot) = (\cdot)'' + 2n r^{-1}(\cdot)' - l_n(\cdot), \ l_n(\cdot) = (n - n^2) r^{-2}(\cdot), \ s_n(\cdot) = \tag{2.5}$$
$$r^{-1}(\cdot)' + l_n(\cdot), \ m_n(\cdot) = r^{-1}(\cdot) - \sigma_n(\cdot),$$
$$\beta_n(1) = \beta_n'(1) = 0, \ \lim_{r\to 0} |\beta_n' r^{-1}| < \infty, \ \lim_{r\to 0} |\beta_n''' r^{-1}| < \infty, \ \beta_n = \gamma_n, \delta_n.$$

The derivation of the branching equation was presented in [5]. Some of its features were explained in [4]. In the notation we have adopted, it has the form

$$G_n(\xi_n, \delta, \tau_n) \equiv 4\xi_n^3 L_{30}^n + \tau_n L_\tau^n + 4\xi_n \delta L_{11}^n + \ldots = 0,$$
$$L_{11}^n = \int_0^1 g_n'(r) S(r)\,dr, \quad L_\tau^n = \mu^q \int_0^1 f_n(r) D_n(\omega_n, \varphi_n, \omega, f, r)\,dr,$$
$$L_{30}^n = \frac{1}{2} \int_0^1 \Big\{ r^{4n+1} \left[\gamma_n \alpha_n(\omega_n, \varphi_n) + \frac{1}{2} \delta_n \alpha_n(\omega_n, \omega_n) \right] -$$

$$-\psi_n\Omega_n(\omega_n, \omega_n) - 2g_n\Omega_n(\omega_n, \varphi_n)\Big\} dr, \;\; H_d(\cdot) = d\vartheta_n(\cdot) + d'\sigma_n(\cdot),$$

$$S(r) = \int_0^r \left(\int_0^t t'\eta(t')\,dt'\right) dt, \;\; D_n(\omega_n, \varphi_n, \omega, f, r) = r^n [H_f\omega_n + H_\omega\varphi_n - n^2(\omega_n f' + \varphi_n\omega')]. \tag{2.6}$$

The structure of quasi-polynomial (2.6) and the explicit expression for the functional L_{11}^n make it possible to isolate the bifurcatively unstable modes of equilibrium. In fact, let μ, θ, and n be fixed and $\tau_n = 0$, $p \in \{p_n\}$. The minor solution $\xi_n(\delta)$ will be bifurcatively unstable if the equation $G(\xi_n, \delta, 0) = 0$ is solvable at $\delta > 0$ for some disturbances S(r) and solvable at $\delta < 0$ for other disturbances. In physical terms, this definition means that the type of branching and the mechanical phenomena that accompany it depend for unstable modes on the distribution of the external load. It follows from the application of Poiseuille's theorem to (2.6) that for such a property to exist, it is sufficient that there be a change in the sign of the coefficient L_{11}^n for different $S_n(r)$. It should be noted that, in accordance with the method of its derivation, the quantity L_{11}^n corresponds to the work of the external force for the minor equilibrium mode. Thus, it is natural to consider as stable those solutions $\xi_n(\delta)$ whose neighborhood does not depend on the sign of the work variation. The criterion of branching instability is evident from this and the properties of the functional L_{11}^n.

We introduce the set $B_n^+ = \{r | g_n'(r) > 0\}$, $B_n^- = \{r | g_n'(r) < 0\}$. If $B_n^+, B_n^- \neq \varnothing$, then the minor solution $\xi_n(\delta)$ is unstable. Thus, to establish this property for the solution of Marguerre–Vlasov problem (1.1), it is sufficient to integrate system (2.4) and check to see whether or not the function $g_n'(r)$ is sign-changing.

Let $\tau_n \neq 0$. In accordance with [5], the sign of the δ-neighborhood of $\xi_n(\delta)$ for a spherical shell with sufficiently small geometric flaws is independent of these disturbances. Meanwhile, the function sign δ has one sign at $\tau_n = 0$ and $\tau_n \neq 0$, $|\tau_n| \ll 1$. However, the study of Eq. (2.6) is considerably more complex at $\tau_n \neq 0$, since it contains two independent parameters δ and τ_n.

Using the Weierstrass approximation theorem, we construct the below Weierstrass polynomial for $G_n(\xi_n, \delta, \tau_n)$:

$$\xi_n^3 + H_2^n\xi_n^2 + H_1^n\,\xi_n + H_0^n = 0,$$
$$H_0^n = (1/4)L_\tau^n (L_{30}^n)^{-1}\tau_n + \ldots, \;\; H_1^n = L_{11}^n (L_{30}^n)^{-1}\delta + \ldots, \;\; H_2^n = O(\delta) + \ldots \tag{2.7}$$

Here, only the dominant terms are presented for the analytic function $H_k^n(\delta, \tau_n)$ (k = 0, 1, 2).

We pick out the values of δ and τ_n that lie on the discriminant line. In order to do this, the Poincaré–Lyapunov parameter must simultaneously make identical the Weierstrass polynomial and the equation

$$3\xi_n^2 + 2H_2^n\xi_n + H_1^n = 0. \tag{2.8}$$

Let us obtain the resultant $\mathcal{R}(\delta, \tau_n)$ for (2.7)-(2.8). Using the Newton diagram method and the Poiseuille theorem, we obtain from the requirement $\mathcal{R}(\delta, \tau_n) = 0$ the perturbation of the eigenvalue of the flawed shell

$$\delta = -\frac{3}{4}\frac{L_{30}^n}{L_{11}^n}(\tau_n L_\tau^n / L_{30}^n)^{2/3} + O(\tau_4^{4/3}), \;\; L_{11}^n \neq 0, \;\; L_{30}^n \neq 0. \tag{2.9}$$

Several conclusions follow from (2.9).

In the sequence $\{p_n\}$, ordered with respect to the value of the p-coordinates of the branch point, it is possible to have a redistribution occur with a change in the type of bifurcation if at the beginning of the spectrum the function $\mathrm{sign}(L_{30}^m L_{11}^m)$ is greater than zero for some n = m and the function $\mathrm{sign}(L_{30}^k L_{11}^k)$ is less than zero for other n = k. In the first case, $\delta < 0$. Thus, p_m, corresponding to the eigenfunction having the rotation group C_m, undergoes a decrease. In the second case, p_k increases. When crowding occurs in $\{p_n\}$ due to a singular perturbation, this leads to a situation whereby the sign of the inequality $p_m > p_k$ may be reversed by a suitable choice of τ_m and τ_k. In this case, the amplitudes of the geometric flaws turn out to be very small, since we are dealing here only with those elements from $\{p_n\}$ that satisfy the estimate

$$|p_m - p_k| < C_{k,m}\mu, \;\; \mu \ll 1 \tag{2.10}$$

($C_{k,m}$ are constants independent of μ, $C_{k,m} \ll 1$).

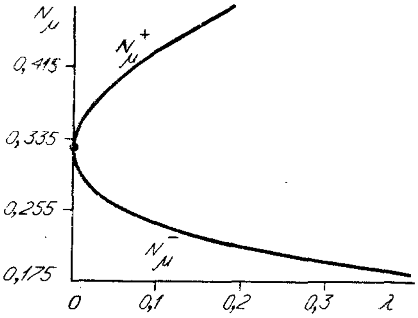

Fig. 1

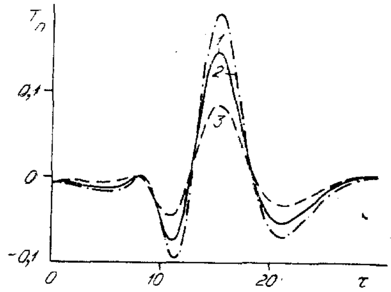

Fig. 2

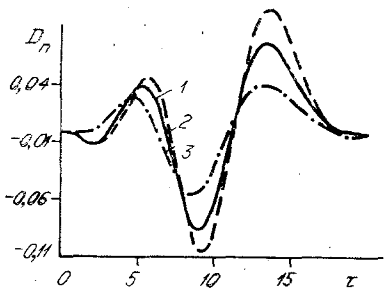

Fig. 3

Thus, it is clear that the change in sign near p* for the function $\text{sign}(L_{30}^{n}L_{11}^{n})$ must be considered when studying the mechanism of instability. If L_{30}^{n} and L_{11}^{n} are of fixed sign for the first elements of the sequence $\{p_n\}$, then we reason as we did above and conclude that redistribution in $\{p_n\}$ also occurs with sufficiently large geometric flaws. In a new sequence of branch points $\{p_{\ell}^{!}\}$ ordered with respect to the p-coordinate, there is a change in the correspondence between the number of the eigenvalue and the group rotation of the eigenfunction. However, unlike before, in this case there is no change in the type of branching of the minor equilibrium modes.

It is evident that the quantitative characteristics of the difference between $\{p_n\}$ and $\{p_{\ell}^{!}\}$ depend on $f_n(r) \in \mathscr{F}_\tau(\Omega)$.

3. An approximate analysis of boundary-value problems performed by W (2.2), (2.3) by the variational method [6-8] with q = 1, k = 2. For $\mu < \mu_0$, $\mu_0 \ll 1$, we replaced (ω, f) in the calculations by the zeroth terms of an expansion in integral powers of μ [9]

$$\omega(r, \mu) \sim \sum_{i=0}^{j} \mu^i [\omega_i(r) + \Pi_i\omega(\tau)],$$

$$f(r, \mu) \sim \sum_{i=0}^{j} \mu^i [f_i(r) + \Pi_i f(\tau)], \quad \mu\tau = 1 - r.$$

Here, $(\cdot)_i$ and $\Pi_i(\cdot)$ are constructed by using the first and second steps in the method of boundary-layer functions. The existence of the asymptote is proven on the basis of the Newton–Kantorovich method if segments of the series at $7 \leqslant j < \infty$ [10] are taken as the initial approximation. The required a priori estimates were obtained only for $3\mu < \theta^2 p$. This limitation and the results calculated for different μ allowed us to take 10^{-4} for μ_0. The asymptotic expansions were substantiated and the corresponding calculations were performed in accordance with a well-established method. Nonetheless, the question of their validity in (1.1) remains unanswered. The reason for this is the difficulty of assigning the order of the mechanical quantities in (1.2) at $\mu \to 0$. The choice k = 2 is almost obvious [4], but it is difficult to determine the order of the nonlinear terms. The fact that q = 1 is of interest in regard to substantiation of the asymptote becomes evident only in the course of the calculations [6, 7]. No axisymmetric branch points were found for other k and q.

Equations (2.3)-(2.5) were analyzed by the method of orthogonal trial run [4, 11]. To improve the stability of the calculation, the Schmidt orthogonalization process was replaced in [4] by calculation of the basis with transformations of the reflection. A description of this modification of the trial run method was given in [11] (S. K. Godunov pointed out the expediency of this approach to the author).

Figure 1 shows the distribution function of the nonaxisymmetric branch points. Here and below, $\theta = 0.15$, $\mu = 10^{-6}$, n = 0.3. The function was introduced by means of the formula

$$\begin{gathered} N_\mu(\lambda) = \mu(\lambda_{k+1} - \lambda_k)^{-1}[\lambda + k\lambda_{k+1} - (k+1)\lambda_k], \\ \lambda \in [\lambda_k, \lambda_{k+1}],\ k = n, n \pm 1, n \pm 2, \ldots, \lambda_k = p_k/p^* - 1. \end{gathered} \tag{3.1}$$

In accordance with (3.1), the only values of λ having mechanical significance are those for which the equality $\text{Entier}(\mu^{-1}N_\mu(\lambda)) = \mu^{-1}N_\mu(\lambda)$ is satisfied. It is evident that $N_\mu(\lambda)$ has two branches $N_\mu^{+}(\lambda)$, $N_\mu^{-}(\lambda)$ and that $\lambda = 0$ – the beginning of the spectrum – is a branch point. A small value was chosen for μ so that, within a practicable range for convergence, $N_\mu(\lambda)$ would coincide with $N_0(\lambda)$, where $N_0(\lambda) = \lim\limits_{\mu \to 0} N_\mu(\lambda)$. Thus, $N_\mu(\lambda)$ in Fig. 1 gives the asymptote of the spectrum of critical pressures. Evaluating it by means of the constant in (2.10), it is easily seen that $C_{k,m} \to \min_{k,m}$ at $\lambda \to 0$. This in turn means that p^* is a condensation point of the sequence $\{p_n\}$.

Figure 2 shows $T_n(\tau) = 10g'_n(\tau)$ for $N_\mu = 0.329$, $N^+_\mu = 0.385$, $N^-_\mu = 0.245$ (lines 1-3). We used the normalization condition $\|\omega_n\|_C = \mu$ in every case. Since both multipliers B^+_n and B^-_n are nontrivial, then we can conclude on the basis of the proposed criterion that the minor equilibrium mode corresponding to each curve in Fig. 2 is bifurcatively unstable. This mode exists at $\delta > 0$ for some disturbances $S(r)$ and at $\delta < 0$ for other disturbances $S(r)$.

We introduce the new variable $\tau = \mu^{-1}(1 - r)$ into $f_n(r)$. We will regard the functional L^n_τ) from (2.6) as a Fredholm integral operator assigned for the elements $f_n(\tau)$. Figure 3 shows its kernel $D_n(\tau)$ for $n = \text{Entier}(\mu^{-1}N_\mu)$, $N_\mu = 0.329, 0.385, 0.315$ (curves 1-3); $D_n(\tau)$ is localized at $\tau < 20$. This inequality indicates the size of the region checked for imperfections in a thin shell. The existence of extrema in $D_n(\tau)$ determines the values of τ for which the change in the form of the middle surface produces the largest change in the disturbance of critical pressure with a fixed amplitude of τ_n. Figure 4 shows the law governing redistribution in the spectrum of the flawed shell. The curves were plotted with $\eta(r) = \text{const}$ and $L^n_\tau = \sup\limits_{f_n(r)} \int_0^1 f_n(r) D_n(\omega_n, \varphi_n, \omega, f, r)dr, \quad f_n \in C$. Here, $y = 10^2(\lambda_n + \delta/p^*)$, $\mu x_n = 10^3\tau_n$, and, as before, the rotation group C_n of the eigenfunction is determined through $N_\mu = 0.335$, 0.329, 0.325, 0.295, 0.305 (lines 1-5). The curves characterize the method of loss of stability. If y lies on the dashed curve, then branching of the mode of equilibrium is accompanied by rupture of the shell. Otherwise, buckling takes place. We will first restrict ourselves to two discriminants (curves 2 and 5). At $x_n = 0$, the minimum branch point lies at the point (0, 0). An increase in x_n is accompanied by an increase in one eigenvalue and a decrease in the other. If x_n is equal to the abscissa of point a, then the critical pressure will be twice degenerate. Further displacement of x_n to the right leads to a situation whereby the first eigenvalue again becomes simple, but there is a change in C_n and the mode of instability. The presence of two discriminants may increase the maximum multiplicity of the degeneracy. In particular, the latter reaches five for disturbances for which x_n is equal to the abscissa of points a, b, c, and d. Generally speaking, multiple critical pressures exist when the functionals of Eq. (2.6) and the amplitudes of the geometric flaws satisfy the condition

$$p + \delta\left(L^n_{30}, L^n_{11}, L^n_\tau, \tau_n\right) = \text{const} \tag{3.2}$$

simultaneously for several n.

Above, we examined only one limiting property of nonaxisymmetric branching. In the other case, we can assign $w_\tau(r, \varphi) \in \mathscr{F}_\tau(\Omega)$, so that no other point except the first will exist in a certain neighborhood of the beginning of the spectrum $\{p_n\}$. In fact, for the sake of definiteness we will take two similar elements p_ℓ and p_k from $\mu\ell = 0.329$, $\mu k = 0.335$. It is evident that two sufficiently large nonintersecting segments of the parameter p exist for p_ℓ and p_k if the condition $|\tau_k| \gg |\tau_l|$ is satisfied for the geometric flaws of the middle surface.

Let us summarize the results.

1. A thin flawed spherical shell becomes unstable due to rupture.

2. By varying the distribution of the geometric flaws and the amplitudes, we can use (2.9) to model the range of cases of loss of stability if crowding occurs in the critical-pressure spectrum of the perfect shell.

The expediency of accounting for this effect in the development of the branching theory of stability has been discussed repeatedly in [12-17].

In accordance with current representations, the supercritical deformation of a thin spherical shell corresponds to a chain of successive branchings. Meanwhile, the wide range of modes of instability is related not so much to the number of branch points in an individual chain as it is to the possibility of there being a large number of such chains [8]. In connection with this, redistribution at the beginning of the spectrum is of particlar interest, since it reveals the mechanism of "creation" of the first point in the above-described model of instability.

3. Branching of the solution without degeneracy remains typical for small $\mu > 0$. To increase the multiplicity of the critical pressure, it is necessary that condition (3.2) be satisfied.

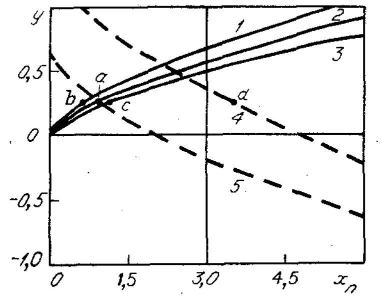

Fig. 4


Fig. 5

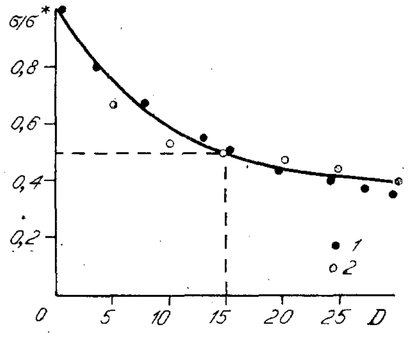

Fig. 6

It should be noted that the radii of the middle surface basically did not figure into our discussions at all, while the main formula (2.9) does not contain parameters of the shell in explicit form. Numerical values of the coefficients of the branching equation are needed to concretize the above-described method of choosing the geometric flaws that change the eigenvalues to the prescribed values. Discussed below is one possible application of the method based on the use of Eq. (2.9).

4. We will examine the stability of a cylindrical shell compressed by an axial force σ and subjected to local thermal shock in the region Ω_0. Figure 5 shows the results of an experiment for a target made of steel Kh18N9T. The thickness of the shell $h = 10^{-3}$ m. At its ends $r = R$, $\varphi \in [0, 2\pi]$, $z = 0$, L we adopted conditions of rigid fastening $w = w_z' = 0$, $v = u_\varphi' = 0$, where w, v, and u are the normal and tangential displacements in the directions e_φ and e_z, respectively; (R, φ, z) are the coordinates of the cylindrical system. One distinctive feature of the tests was the range of variation of the energy flux and the time of action of the radiation. The unit we used made it possible to achieve 1 kJ for the former quantity and a value on the order of 10^{-3} sec for the latter quantity. The energy flow in the problem had a nonuniform (pitch) structure with one pitch lasting 10^{-1} sec. It is known that the results of such local thermal loads are determined by the power density Q. On the whole, in the case we are considering the value of Q was greater than its critical value Q*, which is associated with the beginning of phase transformations in the target within a certain region $\Omega' \subset \Omega_0$. The physical processes accompanying these transformations were described in [19]. The method used to conduct the experiment was described in greater detail in [20, 21].

Figure 5 shows the characteristic rhomboid mode of instability, the region Ω_0 near the common edge, and the irregular boundary $\partial\Omega'$. The following formulation is interesting for such a problem. Let the parameters of the local heat flux and the thin shell of revolution be given. We need to determine the dependence of the branch point on the diameter D of the region Ω_0.

Due to the obvious complexity of modeling the physicomechanical phenomena which take place here, we will use the theory of initial supercritical strain to evaluate the critical force δ_σ caused by the action of the radiation. We will refine δ_σ by specially choosing the imperfections of the middle surfce. Specifically, we continuously extend the Marguerre–Vlasov system into the thermal-shock region Ω_0 so that the shell occupies the simply-connected region $\Omega \times L$ instead of a doubly-connected region. We represent its geometric flaws in the form of the sum $w_\tau(z, \varphi) + W_\tau(z, \varphi)$. Each term has the same rotation group as the first natural mode of a perfect shell whose edges are rigidly fastened. The Young's modulus, Poisson's ratio, and geometric parameters L/R, h/R of the perfect shell coincide with the analogous values for the shell subjected to electrophysical loading. We further assume that $W_\tau(z, \varphi)$ is nontrivial only in a band of width D containing Ω_0 and that the dependence of $w_\tau(z, \varphi)$ on z is described by the indicated eigenfunction. With allowance for these assumptions, we have [5, 22]

$$\delta_\sigma^{3/2} = \frac{3}{2} l \sqrt{-3b^n} (\tau_n + CKD) \sigma/\sigma^*, \quad \delta_\sigma = 1 - l\sigma/\sigma^*,$$
$$b^n \equiv -L_{30}^n / L_{11}^n = -0.827. \tag{4.1}$$

Here, σ is the critical force for the shell subjected to electrophysical loading; $\sigma^* = \min_n\{\sigma_n\}$; σ_n is the point of the cylindrical shell with $w_\tau(z, \varphi) \equiv 0$, $W_\tau(z, \varphi) \equiv 0$; $\tau_n = h^{-1}\|w_\tau\|_c$; C is a constant determined by the type of geometric imperfection of the region Ω_0; K is a free parameter; ℓ is the ratio of the bifurcative force measured in the experiment to its value for the perfect cylindrical shell in the absence of loading.

Figure 6 compares theoretical and experimental forces for L/R = 2. The value of σ obtained from Eq. (2.9) is designated by points 1, while the data obtained from direct measurements is shown by the line and points 2. The parameter K was determined from the condition of agreement of the theoretical value of σ and the experimental value with D = 15 $m \cdot 10^{-3}$. Here, it was found that K = 2.71 at $\tau_n = 0.332$, C = 0.0196, and $\ell = 0.474$. It is evident that the branch points can be predicted by the methods of eigenvalue perturbation theory with the use of data from just one control experiment involving determination of the free parameter K of discriminant curve (4.1).

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